

**ADVANCED SUBSIDIARY GCE
MATHEMATICS**

4732/01

Probability & Statistics 1

TUESDAY 15 JANUARY 2008

Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **6** printed pages and **2** blank pages.

- 1 (i) The letters A, B, C, D and E are arranged in a straight line.
- (a) How many different arrangements are possible? [2]
- (b) In how many of these arrangements are the letters A and B next to each other? [3]
- (ii) From the letters A, B, C, D and E, two different letters are selected at random. Find the probability that these two letters are A and B. [2]

2 A random variable T has the distribution $\text{Geo}\left(\frac{1}{5}\right)$. Find

- (i) $P(T = 4)$, [2]
- (ii) $P(T > 4)$, [2]
- (iii) $E(T)$. [1]

3 A sample of bivariate data was taken and the results were summarised as follows.

$$n = 5 \quad \Sigma x = 24 \quad \Sigma x^2 = 130 \quad \Sigma y = 39 \quad \Sigma y^2 = 361 \quad \Sigma xy = 212$$

- (i) Show that the value of the product moment correlation coefficient r is 0.855, correct to 3 significant figures. [2]
- (ii) The ranks of the data were found. One student calculated Spearman's rank correlation coefficient r_s , and found that $r_s = 0.7$. Another student calculated the product moment coefficient, R , of these ranks. State which one of the following statements is true, and explain your answer briefly.
- (A) $R = 0.855$
- (B) $R = 0.7$
- (C) It is impossible to give the value of R without carrying out a calculation using the original data. [2]
- (iii) All the values of x are now multiplied by a scaling factor of 2. State the new values of r and r_s . [2]

4 A supermarket has a large stock of eggs. 40% of the stock are from a firm called Eggzact. 12% of the stock are brown eggs from Eggzact.

An egg is chosen at random from the stock. Calculate the probability that

- (i) this egg is brown, given that it is from Eggzact, [2]
- (ii) this egg is from Eggzact and is not brown. [2]

- 5 (i) 20% of people in the large town of Carnley support the Residents' Party. 12 people from Carnley are selected at random. Out of these 12 people, the number who support the Residents' Party is denoted by U .

Find

(a) $P(U \leq 5)$, [2]

(b) $P(U \geq 3)$. [3]

- (ii) 30% of people in Carnley support the Commerce Party. 15 people from Carnley are selected at random. Out of these 15 people, the number who support the Commerce Party is denoted by V .

Find $P(V = 4)$. [3]

- 6 The probability distribution for a random variable Y is shown in the table.

y	1	2	3
$P(Y = y)$	0.2	0.3	0.5

- (i) Calculate $E(Y)$ and $\text{Var}(Y)$. [5]

Another random variable, Z , is independent of Y . The probability distribution for Z is shown in the table.

z	1	2	3
$P(Z = z)$	0.1	0.25	0.65

One value of Y and one value of Z are chosen at random. Find the probability that

(ii) $Y + Z = 3$, [3]

(iii) $Y \times Z$ is even. [3]

- 7 (i) Andrew plays 10 tennis matches. In each match he either wins or loses.

(a) State, in this context, two conditions needed for a binomial distribution to arise. [2]

(b) Assuming these conditions are satisfied, define a variable in this context which has a binomial distribution. [1]

- (ii) The random variable X has the distribution $B(21, p)$, where $0 < p < 1$.

Given that $P(X = 10) = P(X = 9)$, find the value of p . [5]

- 8 The stem-and-leaf diagram shows the age in completed years of the members of a sports club.

Male		Female
8 8 7 6	1	6 6 6 7 7 8 8 9
7 6 5 5 3 3 2 1	2	1 3 3 4 5 7 8 8 9 9
9 8 4 4 3	3	2 3 3 4 7
5 2 1	4	0 1 8
9 0	5	0

Key: 1 | 4 | 0 represents a male aged 41 and a female aged 40.

- (i) Find the median and interquartile range for the males. [3]
- (ii) The median and interquartile range for the females are 27 and 15 respectively. Make two comparisons between the ages of the males and the ages of the females. [2]
- (iii) The mean age of the males is 30.7 and the mean age of the females is 27.5, each correct to 1 decimal place. Give one advantage of using the median rather than the mean to compare the ages of the males with the ages of the females. [1]

A record was kept of the number of hours, X , spent by each member at the club in a year. The results were summarised by

$$n = 49, \quad \Sigma(x - 200) = 245, \quad \Sigma(x - 200)^2 = 9849.$$

- (iv) Calculate the mean and standard deviation of X . [6]

- 9 It is thought that the pH value of sand (a measure of the sand's acidity) may affect the extent to which a particular species of plant will grow in that sand. A botanist wished to determine whether there was any correlation between the pH value of the sand on certain sand dunes, and the amount of each of two plant species growing there. She chose random sections of equal area on each of eight sand dunes and measured the pH values. She then measured the area within each section that was covered by each of the two species. The results were as follows.

	Dune	A	B	C	D	E	F	G	H
	pH value, x	8.5	8.5	9.5	8.5	6.5	7.5	8.5	9.0
Area, y cm ² , covered	Species P	150	150	575	330	45	15	340	330
	Species Q	170	15	80	230	75	25	0	0

The results for species P can be summarised by

$$n = 8, \quad \Sigma x = 66.5, \quad \Sigma x^2 = 558.75, \quad \Sigma y = 1935, \quad \Sigma y^2 = 711\,275, \quad \Sigma xy = 17\,082.5.$$

- (i) Give a reason why it might be appropriate to calculate the equation of the regression line of y on x rather than x on y in this situation. [1]
- (ii) Calculate the equation of the regression line of y on x for species P , in the form $y = a + bx$, giving the values of a and b correct to 3 significant figures. [4]
- (iii) Estimate the value of y for species P on sand where the pH value is 7.0. [2]

The values of the product moment correlation coefficient between x and y for species P and Q are $r_P = 0.828$ and $r_Q = 0.0302$.

- (iv) Describe the relationship between the area covered by species Q and the pH value. [1]
- (v) State, with a reason, whether the regression line of y on x for species P will provide a reliable estimate of the value of y when the pH value is
- (a) 8, [1]
- (b) 4. [1]
- (vi) Assume that the equation of the regression line of y on x for species Q is also known. State, with a reason, whether this line will provide a reliable estimate of the value of y when the pH value is 8. [1]

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Note: “(3 sfs)” means “answer which rounds to ... to 3 sfs”. If correct ans seen to ≥ 3 sfs, ISW for later rounding
 Penalise over-rounding only once in paper.

1ia	$5!$ or 5P_5 = 120	M1 A1 2	
b	$4!$ or 4P_4 seen $4! \times 2$ 48	M1 M1dep A1 3	or $2 \times 3!$ or $2! \times 3!$ or $2! \times {}^3P_3$ $2 \times 3! \times 4$
ii	${}^1/5C_2$ or ${}^1/5 \times {}^1/4 \times 2$ or 0.4×0.25 or ${}^2/_{5P2}$ = ${}^1/_{10}$	M1 A1 2	Allow M1 for 3C_2 or ${}^1/5 \times {}^1/4$ or ${}^1/_{20}$ or ${}^1/5 \times {}^1/5 \times 2$ or ${}^2/_{25}$ oe
Total		7	
2i	$({}^4/5)^3 \times ({}^1/5)$ oe = ${}^{64}/_{625}$ or 0.102 (3 sfs)	M1 A1 2	Allow M1 for $({}^4/5)^4 \times ({}^1/5)$
ii	$({}^4/5)^4$ alone or $1 - ({}^1/5 + {}^4/5 \times {}^1/5 + ({}^4/5)^2 \times {}^1/5 + ({}^4/5)^3 \times {}^1/5)$ = ${}^{256}/_{625}$ or 0.410 (3 sfs)	M1 A1 2	Allow $({}^4/5)^3$ or $({}^4/5)^5$; not $1 - ({}^4/5)^4$ Allow one term omitted or wrong or “correct” extra
iii	5	B1 1	
Total		5	
3i	$r = \frac{212 - \frac{24 \times 39}{5}}{\sqrt{(130 - \frac{24^2}{5})(361 - \frac{39^2}{5})}}$	B2 2	$\frac{24.8}{\sqrt{14.8 \times 56.8}}$ or $\frac{24.8}{\sqrt{840.64}}$ or $\frac{24.8}{3.85 \times 7.54}$ or $\frac{24.8}{29}$ B2 for correct subst in r B1 for correct subst in any S
ii	$R = 0.7$ or (B) Definition of r_s is PMCC for ranks	B1 B1 2	(A) and (B) true: B0B0 dep 1 st B1
iii	$r = 0.855$ $r_s = 0.7$	B1 B1 2	or “unchanged”: B1B1 Interchanged: B1
Total		6	
4i	$0.4 \times p = 0.12$ or ${}^{0.12}/_{0.4}$ or ${}^{12}/_{40}$ oe $p = 0.3$ oe	M1 A1 2	
ii	$0.4 \times (1 - \text{their } 0.3)$ oe eg ${}^{40}/_{100} \times {}^{28}/_{40}$ 0.28 or 28% oe	M1 A1ft 2	or $0.4 - 0.12$ or 0.28 or 28 seen Not 0.4×0.88 unless ans to (i) is 0.12
Total		4	
5ia	Binomial stated or implied 0.9806	B1 B1 2	by use of tables or $0.2^a \times 0.8^b$, $a+b = 12$
b	0.5583 seen $1 - 0.5583$ = 0.442 (3 sfs)	M1 M1 A1 3	add 10 corr terms or 1-(add 3 corr terms): M2 or $1 - 0.7946$ or 0.205 or $1 - 0.6774$ or 0.323 or $1 - 0.3907$ or 0.609 or add 9 terms or 1-(add 2 or 4 terms): M1
ii	${}^{15}C_4 \times 0.3^4 \times 0.7^{11}$ = 0.219 (3 sfs)	M2 A1 3	${}^{15}C_4 \times 0.3^{11} \times 0.7^4$: M1
Total		8	

6i	Σyp $= 2.3$ $\Sigma y^2 p$ (= 5.9) $-(\Sigma yp)^2$ $= 0.61$ oe	M1 A1 M1 M1 A1 5	≥ 2 terms added $\div 3$ or $\div 6$ etc: M0 ≥ 2 terms added $\div 3$ or $\div 6$ etc: M0 dep +ve result $(-1.3)^2 \times 0.2 + (-0.3)^2 \times 0.3 + 0.7^2 \times 0.5$: M2 one term correct: M1 Use of Z: MR, lose last A1 (2.55, 0.4475)
ii	$0.2 \times 0.25 + 0.3 \times 0.1$ or $0.05 + 0.03$ alone $= 0.08$ oe	M2 A1 3	M1 for one product eg correct $\times 2$: M1 or clearly ident (1,2), (2,1): M1
iii	$0.3 \times 0.1 + 0.3 \times 0.25 + 0.3 \times 0.65$ $+ 0.25 \times 0.2 + 0.25 \times 0.5$ alone or $0.03 + 0.075 + 0.195 + 0.05 + 0.125$ $= 0.475$ or $^{19}/_{40}$ oe	M2 A1 3	M1 : any 3, 4 of these prods alone or these 5 prods plus 1 extra or repeat or (ii) + prod or $0.3 + \text{prod}$ or $0.25 + \text{prod}$ or clearly identify (1,2) (3,2) (2,2) (2,1) (2,3) M2 for $0.3 + (0.2 + 0.5) \times 0.25$ or $0.25 + (0.1 + 0.65) \times 0.3$ or $0.3 + 0.25 - 0.3 \times 0.25$ or $1 - (0.2 + 0.5)(0.1 + 0.65)$ M1 for $(0.2 + 0.5)(0.1 + 0.65)$
Total		11	
7ia	Results or matches are indep Prob of winning is constant	B1 B1 2	allow "wins" indep; not "trials" indep not "success"
ib	No of wins (or losses)	B1 1	
ii	${}^{21}C_{10} p^{10} q^{11} = {}^{21}C_9 p^9 q^{12}$ $\frac{12}{10} p = q$ or $\frac{12p(1-p)^{-1}}{10} = 1$ or similar $1.2p = 1 - p$ oe eg $p = 0.833(1-p)$ or $352716p = 293930(1-p)$ $p = \frac{5}{11}$ or 0.455 (3 sfs) oe	M1 M1M1 M1 A1 5	or $(1-p)$ for q & allow omit bracket or $352716p^{10}q^{11} = 293930p^9q^{12}$ M1 for $^{12}/_{10}$ or $^{6}/_5$ or 1.2 or $^{5}/_6$ or 0.833 M1 for p & q cancelled correctly or equiv equn in p or q (cancelled) nos not nec'y cancelled; not alg denom
Total		8	

